Simple Probability

Inquire: A 20% Chance of Rain

Overview

The weatherman said it might rain today. There's a 20% chance. That means there's an 80% chance it won't. The probability is that it won't rain. Probability tells us how often an event is likely to occur. It is frequently given as a percentage, but sometimes as a ratio. For example, the odds are 1 to 5 that it will rain today.

We all talk about odds and make predictions, but usually our predictions are based only on our experiences and feelings. This is called subjective probability. However, when a weatherman predicts rain, he isn't basing his forecast on his feelings. He has computers analyzing complex weather patterns to determine the probability of rain. Probability is based on the number of desired or favorable outcomes divided by the number of possible outcomes. If you know those two numbers, you can figure probability with confidence, and you will probably be right!

Big Question: Why might it be important for you to know the future probability predictions for salaries in your profession?

Watch: Probability is Everywhere

What role does probability play in your daily life? It is probably a bigger role than you realize. From insurance rates to weather forecasts, someone somewhere makes predictions that affect your life. The grocer predicts how much lettuce you will buy. The restaurant owner figures the probability that you will come in and order his special on Tuesday. The insurance company bets you won't have an accident and you bet you will!

Probability tells us how often an event is likely to occur. The formula is: \( P(E) = \frac{n(E)}{n(S)} \). In other words, the probability that an event will occur is equal to the number of favorable outcomes divided by the number of possible outcomes. Say you are going to roll one die, and you want to roll a four. The probability of that event occurring is \( \frac{1}{6} \). There is only one 4 on a die, but there are six outcomes possible. This could also be expressed as 16.6%. There is a 16.6% chance that in one roll of one die, the outcome will be a 4.

All probability is expressed in numbers between, and including, zero and one. A probability of zero means there is no possibility that an event will occur. A probability of one means an event will definitely occur. Probability can be expressed as a ratio, a fraction, or a percentage. The important thing to remember is that if your answer is a negative number or a number greater than one, check your work again.
Insurance companies use finely-tuned probability tables on every type of policy. Age and gender are considered on auto insurance, because some groups have more accidents than others. The driver’s geographic location and miles driven per year are also entered into the equation. Health insurance rates are higher for smokers, because they have a greater probability of health problems. In fact, there are probability statistics behind the cost of most things you purchase.

It gets complicated, but you don’t have to work it all out yourself. There are analysts who crunch these numbers daily. You just need an awareness of the science and math behind the business of living your daily life. Probability has probably touched everything you touched today.

Read: Simple Probability

Overview

It is often necessary to "guess" the outcome of an event in order to make a decision. Politicians study polls to guess their likelihood of winning an election. Teachers choose a particular course of study based on what they think students will understand. Doctors choose the treatments needed for various diseases based on their assessments of likely results. People play casino games because they believe the likelihood of winning is good. You may have even chosen your course of study based on the probable availability of jobs.

You have, more than likely, used probability. In fact, you probably have an intuitive sense of probability. Probability deals with the chance of an event occurring. Whenever you weigh the odds of whether or not to do your homework or study for an exam, you are using probability. In this lesson, you will learn how to solve probability problems.

Defining Probability

Probability is the chance that something will or will not happen.

Let P = Probability    S = Sample Space    E = Event

1. $P(∅) = 0$    The probability of an empty event happening is zero. The smallest possible probability of an event happening is zero. This is also sometimes written in set notation as $∅$ (null set or empty set.)

2. $P(S) = 1$    The probability of the total value of the sample space happening is 1. The largest possible probability of a certain event is 1.

3. $0 \geq P(E) \leq 1$    The probability of an event occurring is greater than or equal to zero and less than or equal to 1. Probabilities exist between 0 and 1, inclusive.

The results of calculating simple probability should always be between zero and one. If the answer is less than zero or greater than one, recheck.

Calculating Probability of Simple Events

The formula for simple probability is $P(E) = \frac{n(E)}{n(S)}$

P = Probability
In other words, the simple probability formula means that the probability of an event occurring is equal to the number of the events possible divided by the number in the sample space.

Probability is a measure that is associated with how certain we are of outcomes of a particular experiment or activity. An experiment is a planned operation carried out under controlled conditions. If the result is not predetermined, the experiment is said to be a chance experiment. Flipping one fair coin twice is an example of an experiment.

A result of an experiment is called an outcome. The sample space of an experiment is the set of all possible outcomes. The uppercase letter $S$ is used to denote the sample space. For example, if you flip one fair coin, $S = \{H,T\}$ where $H$ (heads) and $T$ (tails) are the outcomes.

An event is any combination of outcomes. Uppercase letters like $A$ and $B$ represent events. For example, if the experiment is to flip one fair coin, event $A$ might get at most one head. The probability of an event $A$ is written $P(A)$.

The probability of any outcome is the relative frequency of that outcome. Probabilities are between zero and one, inclusive (that is, zero, one, and all numbers between these values). $P(A) = 0$ means the event $A$ can never happen. $P(A) = 1$ means the event $A$ always happens. $P(A) = 0.5$ means the event $A$ is equally likely to occur or not to occur. For example, if you flip one fair coin repeatedly (from 20 to 2,000 to 20,000 times) the relative frequency of heads approaches 0.5 (the probability of heads).

Equally likely means that each outcome of an experiment occurs with equal probability. For example, if you toss a fair, six-sided die, each face (1, 2, 3, 4, 5, or 6) is as likely to occur as any other face. If you toss a fair coin, a Heads (H) and a Tails (T) are equally likely to occur. If you randomly guess the answer to a true/false question on an exam, you are equally likely to select a correct answer or an incorrect answer.

Suppose you roll one fair six-sided die with the numbers $\{1, 2, 3, 4, 5, 6\}$ on its faces. Let event $E =$ rolling a number that is at least five. There are two outcomes $\{5, 6\}$. $P(E) = 2/6$. If you were to roll the die only a few times, you would not be surprised if your observed results did not match the probability. If you were to roll the die a very large number of times, you would expect that, overall, $2/6$ of the rolls would result in an outcome of "at least five." However, you would not expect exactly $2/6$. The long-term relative frequency of obtaining this result would approach the theoretical probability of $2/6$ as the number of repetitions grew larger and larger. A higher probability expresses a higher degree of certainty that something will happen. An event with a probability of one is certain to happen.

**Subjective Probability**

The word subjective implies opinion and feeling; it is the opposite of objective which is fact based. Therefore, **subjective probability** is not based on facts, but on personal opinions and observations. Because you had trouble finding a parking place at the mall the last three times you went, you figure the probability is high that you'll have the same experience today. Or, because you had trouble the last three times, you figure today will be your lucky day and you will probably find a great spot.
Subjective probability has a very low certainty. It has no formula for calculation. It is based on feeling. One example of subjective probability is called the gambler's fallacy. This is the gambler's belief that he can outwit the casino and win at a particular game. The opinion is usually based on past experience and observation. For example, a gambler might reason that since the roulette wheel has landed on black the past four spins, it is likely to land on black again. Conversely, he might reason that since the wheel landed on black the past four times, it's likely to land on red this time. It's called a fallacy for a reason!

Reflect Poll: Do You Feel Lucky?

Subjective probability is not based on a formula, but on prior experiences and emotions. Some people try to predict future events based on subjective probability. How much would you trust their predictions?

- I know people whose "gut feelings" are right most of the time. I trust them.
- It's not reliable if it hasn’t been proven scientifically. I don’t trust them.
- Sometimes I trust them; sometimes I don’t.

Expand: Solving Simple Probability

Overview

The solution to simple probability problems requires a two- or three-step equation. Remember that the answer is always going to be the number of favorable outcomes divided by the number of possible outcomes. The answer should always be given as a reduced fraction. Since probability can not be less than zero or greater than one, the reduced fraction is the expected answer.

Problem Using One Die

If a die is rolled, what is the probability that an even number is obtained?

Let’s first write the sample space (S) of the experiment:
S = {1,2,3,4,5,6}

Let E = the event (an even number is obtained):
E = {2,4,6}

We now use the formula for probability:
P(E) = \frac{n(E)}{n(S)}
P(E) = \frac{3}{6}
P(E) = \frac{1}{2}

Coin Toss Problem

Two coins are tossed. Find the probability that two heads are obtained. Each coin has two possible outcomes: heads (H) and tails (T).

Let’s first write the sample space (S) of the experiment:
S = {(H,T), (H,H), (T,H), (T,T)}
Let \( E = \) the event (two heads are obtained):
\[
E = \{(H,H)\}
\]

Use the formula for probability:
\[
P(E) = \frac{n(E)}{n(S)}
\]
\[
P(E) = \frac{1}{4}
\]
\[
P(E) = \frac{1}{4}
\]

**Problem Using Two Dice**

Two dice are rolled. Find the probability that the sum is equal to 4.

The sample space \( S \) of two dice is shown below:
\[
S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\}
\]
\[
\{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\}
\]
\[
\{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}
\]
\[
\{(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\}
\]
\[
\{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}
\]
\[
\{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}
\]

Let \( E = \) the event (sum equal to 4). There are three possible outcomes to give a sum of 4: \( E = (1,3), (2,2), (3,1) \).

Use the formula for probability:
\[
P(E) = \frac{n(E)}{n(S)}
\]
\[
P(E) = \frac{3}{36}
\]
\[
P(E) = \frac{3}{36}
\]
\[
P(E) = \frac{1}{12}
\]

Working with simple probability is a matter of always finding the number of desired outcomes and dividing by the number of possible outcomes. Reduce the fraction to the lowest terms. Read the probability by either changing it to a percent \((\frac{1}{2} = 50\% \text{ probability})\) or simply stating that the probability is 1 out of 2.

**Lesson Toolbox**

**Additional Resources and Readings**

A video showing solution examples for simple probability with a die
- Link to resource: https://youtu.be/3wcsXGzF4Bo

A video providing a very detailed explanation of probability
- Link to resource: https://youtu.be/WqTioYM0L7I

A video tutorial on how to solve for the probability of a simple event
- Link to resource: https://youtu.be/zpSE-xQ2gHE

**Lesson Glossary**

**certainty**: expresses how high the probability is that something will happen

**event**: one or more outcomes of an experiment
**experiment:** any activity with an observable result

**gambler's fallacy:** a gambler’s belief that he can outwit a game of chance based on his past experiences and feelings; For example, “That slot machine hasn’t paid off in a long time; it’s about to hit a jackpot!”

**sample space:** the set of all possible outcomes of an experiment

**simple probability:** a number between zero and one, inclusive, giving the likelihood that a specific event will occur

**subjective probability:** probability based on prior experiences and feelings, rather than numbers

**outcome:** a possible result of an experiment

**Check Your Knowledge**

1. Which of these numbers can not be a probability?
   a. -0.000001
   b. 0.5
   c. 0
   d. 1

2. One die is rolled. What is the probability that the number rolled is greater than 4?
   a. ⅓
   b. ¼
   c. ½
   d. ⅕

3. The probability of an event occurring is equal to the number of favorable outcomes __________ the number of possible outcomes.
   a. multiplied by
   b. added to
   c. divided by
   d. subtracted from

**Answer Key:**

**Citations**

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**Adapted Content:**
Link to resource: https://cnx.org/contents/MBiUQmmY@21.2:Mm7i4AzN@8/Introduction

Link to resource: https://cnx.org/contents/tWu56V64@34.98:py_tPsy1@15/Terminology