Angles, Triangles, and Pythagorean Theorem

Inquire: The 3-Dimensional World

Overview

Objects in the 3-dimensional world we live in come in all shapes and sizes. If we want to study them, we need to start with the basic building blocks of 3-D shapes which are 2-dimensional shapes. Every shape is made up of a combination of sides and angles. The most basic of them is the triangle. A triangle has 3 sides and 3 angles. Any smaller than this, and we would have a line, which is considered a 1-dimensional shape. By the end of the lesson, students will apply properties of angles, triangles, and Pythagorean Theorem to find missing angles and sides.

Big Question: What is the relationship between the angles and sides of a triangle?

Watch: Triangles

Triangles are a basic building block for all 2-dimensional and 3-dimensional shapes. If you can understand how triangles are put together, you can extend that knowledge to all other shapes.

For example, if you draw two right triangles of the same size, rotate one of them 180° degrees, then connect them together, you will form a rectangle. This allows us to apply angle or area formulas for triangles to angle or area formulas for rectangles.

Another example uses equilateral triangles. These are triangles that have the same length of all of their sides. If you arrange the tip of six equilateral triangles to touch, you can make a hexagon. If we can find the area of one of those triangles, we can multiply it by 6 to find the area of the whole hexagon.

Right triangles are useful when building basic shapes or studying the relationship between angles and sides. You may have even seen a construction worker using a special, right triangle ruler to measure an object. An important branch of math that uses right triangles to study the relationship between sides and angles of triangles is called trigonometry.

We can use basic shapes to model objects all around us. Look around you, and you might start to see shapes. If you are sitting in a room, you are surrounded by rectangle walls and a ceiling. Maybe you drive a car? The wheels can be modeled using circles. You are looking at a monitor right now. It might be shaped like a rectangle; but, you might notice that when monitors are sold they are measured diagonally. Right triangles can be used to find how long that diagonal is by measuring the length and width of your monitor.
What about 3-dimensional shapes? With four triangles forming a roof and one square forming a base, we can form the mighty Egyptian Pyramids of Giza.

If you are getting a little hungry, we could look for a can of peaches. If you look closely, that can has two circle lids and a curved, rectangle connecting them together to form a cylinder.

Do you like shipping and receiving items from a thrifty online purchase? Then you can appreciate combining six different sized rectangles to form the box the items are shipped in.

Once you start breaking objects down into their basic parts, you can start understanding how we can put objects together. It all starts with our basic building block, the triangle.

Look around you. What are other examples of basic shapes being used to form everyday items?

Read: Angles and Triangles

In the following section, we will cover properties of angles, properties of triangles, properties of similar triangles, and how to apply their properties to find missing sides and angles.

Definition of Angles

Are you familiar with the phrase do a 180°? It means to make a full turn so that you face the opposite direction. It comes from the fact that the measure of an angle that makes a straight line is 180 degrees. See the figure below.

A ray is a line that has one endpoint. An angle is formed by two rays that share a common endpoint. Each ray is called a side of an angle and the common endpoint is called the vertex of an angle (plural is vertices). An angle is named by its vertex. In the figure below, \( \angle A \) is the angle with vertex at point A. The measure of \( \angle A \) is written \( m \angle A \).

We measure angles in degrees, and use the symbol ° to represent degrees. We use the abbreviation \( m \) for the measure of an angle. So if \( \angle A = 27° \), we would write \( m \angle A = 27 \).

Supplementary and Complementary Angles

If the sum of the measures of two angles is 180°, then they are called supplementary angles. In the figure below, each pair of angles is supplementary because their measures add up to 180°. We could also say that each angle is the supplement of the other.
If the sum of the measures of two angles is 90°, then the angles are **complementary angles**. In the figure below, each pair of angles is complementary, because their measures add to 90°. We could say that each angle is the complement of the other.

As we work through applications of geometry, it will be helpful to draw a figure and then label it with the information from the problem.

**Supplementary and Complementary Angle Formulas**

If $\angle A$ and $\angle B$ are supplementary, then $m\angle A + m\angle B = 180°$.

If $\angle A$ and $\angle B$ are complementary, then $m\angle A + m\angle B = 90°$.

**Example 1:** An angle measures 40°. Find A) its supplement, and B) its complement.

First, let's draw and label the information we have.

A) If we want its supplement, let's set up an equation and set it equal to 180°.

$$m\angle A + m\angle B = 180$$

$$s + 40 = 180$$

If $s = 140$, then the supplement of the 40° angle is 140°.
B) If we want its complement, let's set up an equation and set it equal to 90°.

\[ m\angle A + m\angle B = 90 \]
\[ c + 40 = 90 \]
If \( s = 50 \), then the complement of the 40° angle is 50°

Example 2: An angle measures 77°. Find A) its supplement, and B) its complement.

You try drawing and labeling a model, plugging in values in the appropriate formula, and then solving. The appropriate work is below.

A) If we want its supplement, let's set up an equation and set it equal to 180°.

\[ m\angle A + m\angle B = 180 \]
\[ s + 77 = 180 \]
If \( s = 83 \), then the supplement of the 77° angle is 103°

B) If we want its complement, let's set up an equation and set it equal to 90°.

\[ m\angle A + m\angle B = 90 \]
\[ c + 77 = 90 \]
If \( s = 13 \), then the complement of the 77° angle is 13°

Sum of Angles in a Triangle

Triangles have three sides and three angles. Triangles are named by their vertices. The triangle in the figure below is called \( \triangle ABC \), read ‘triangle ABC.’ We label each side with a lower case letter to match the upper case letter of the opposite vertex.

The three angles of a triangle are related in a special way. The sum of their measures is 180°.

\[ m\angle A + m\angle B + m\angle C = 180° \]

Example 3: The measures of two angles of a triangle are 55° and 82°. Find the measure of the third angle.

First, let's draw and label our information. We will say that \( m\angle A = 55 \) and \( m\angle B = 82 \), so we can find our third angle. The letters we pick will not affect our calculations.
Now, let's plug values in the appropriate formula and solve.

\[
m \angle A + m \angle B + m \angle C = 180^\circ
\]
\[
82 + 55 + x = 180
\]
\[
137 + x = 180
\]
If \( x = 43 \), then the measure of the third angle is 43°.

Example 4: The measures of two angles of a triangle are 31° and 128°. Find the measure of the third angle.

You try drawing and labeling a model, plugging in values in the appropriate formula, and then solving. The appropriate work is below.

\[
m \angle A + m \angle B + m \angle C = 180^\circ
\]
\[
31 + 128 + x = 180
\]
\[
159 + x = 180
\]
If \( x = 21 \), then the measure of the third angle is 21°.

Right Triangles

Some triangles have special names. A right triangle has one 90° angle, which is often marked with a square in the angle shown in the figure below.

If we know that a triangle is a right triangle, we know that one angle measures 90° so we only need the measure of one of the other angles in order to determine the measure of the third angle.

Example 5: One angle of a right triangle measures 28°. What is the measure of the third angle?

First, let's draw and label our information. Since this is a right angle, we know that one of the angles is 90°.
Now, let's plug values in the appropriate formula and solve.

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]
\[ 28 + 90 + x = 180 \]
\[ 118 + x = 180 \]
If \( x = 62 \), then the measure of the third angle is \( 62^\circ \).

Example 6: One angle of a right triangle measures \( 56^\circ \). What is the measure of the other angle?

You try drawing and labeling a model, plugging in values in the appropriate formula, and then solving. The appropriate work is below.

\[ m\angle A + m\angle B + m\angle C = 180^\circ \]
\[ 56 + 90 + x = 180 \]
\[ 146 + x = 180 \]
If \( x = 34 \), then the measure of the third angle is \( 34^\circ \).

Reflect Poll: Triangles. Triangles Everywhere.

How surprised are you that triangles can form almost all the 2-D shapes we use today?
- Very surprised. I am excited to know more about it.
- Fairly surprised. It is an interesting fact.
- Not at all. I already knew it.

Expand: Sides of Right Triangles

Investigate

Next, we will discuss squaring and square rooting. We can use them to find missing sides of right triangles using a formula called "Pythagorean Theorem." It will be useful, since there are a lot of applications for right triangles. For this next section, make sure you have a calculator that can do the square root function.

Defining Perfect Squares and Square Roots

When a number \( n \) is multiplied by itself, we can write this as \( n^2 \), which we read aloud as "\( n \)-squared." For example, \( 8^2 \) is read as "8-squared."
We call 64 the square of 8 because $8^2 = 64$. Similarly, 121 is the square of 11, because $11^2 = 121$.

Below is a table of **perfect squares** which is the square of a whole number.

<table>
<thead>
<tr>
<th>Number</th>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>$n^2$</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
</tr>
</tbody>
</table>

Sometimes we will need to look at the relationship between numbers and their squares in reverse. Because $10^2 = 100$, we say 100 is the square of 10. We can also say that 10 is a square root of 100.

Below is the notation for square root and a list of square roots of perfect squares. Notice how they all are whole numbers.

![radical sign \( \sqrt{m} \) radicand]

<table>
<thead>
<tr>
<th>$\sqrt{1}$</th>
<th>$\sqrt{4}$</th>
<th>$\sqrt{9}$</th>
<th>$\sqrt{16}$</th>
<th>$\sqrt{25}$</th>
<th>$\sqrt{36}$</th>
<th>$\sqrt{49}$</th>
<th>$\sqrt{64}$</th>
<th>$\sqrt{81}$</th>
<th>$\sqrt{100}$</th>
<th>$\sqrt{121}$</th>
<th>$\sqrt{144}$</th>
<th>$\sqrt{169}$</th>
<th>$\sqrt{196}$</th>
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<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

**Approximating Square Roots**

There are mathematical methods to approximate square roots, but it is much more convenient to use a calculator to find square roots. Find the $\sqrt()$ or $\sqrt{x}$ key on your calculator. You will to use this key to approximate square roots. When you use your calculator to find the square root of a number that is not a perfect square, the answer that you see is not the exact number. It is an approximation, to the number of digits shown on your calculator’s display. The symbol for an approximation is $\approx$ and it is read approximately.

Suppose your calculator has a 10-digit display. Using it to find the square root of 5 will give 2.236067977. This is the approximate square root of 5. When we report the answer, we should use the “approximately equal to” sign instead of an equal sign. We will typically round this to the nearest 2 decimal places.

$$\sqrt{5} \approx 2.236067978 \approx 2.24$$

**Pythagorean Theorem**

The **Pythagorean Theorem** is a special property of right triangles that has been used since ancient times. It is named after the Greek philosopher and mathematician Pythagoras who lived around 500 BCE.

Remember that a right triangle has a 90° angle, which we usually mark with a small square in the corner. The side of the triangle opposite the 90° angle is called the **hypotenuse**, and the other two sides are called the legs. See the figure below.
The Pythagorean Theorem tells how the lengths of the three sides of a right triangle relate to each other. It states that in any right triangle, the sum of the squares of the two legs equals the square of the hypotenuse.

The Pythagorean Theorem

In any right triangle $\Delta ABC$, $a^2 + b^2 = c^2$

Where $c$ is the length of the hypotenuse, $a$ and $b$ are the lengths of the legs.

Example 1: Use the Pythagorean Theorem to find the length of the hypotenuse.

Since we have the legs, we can say that $a = 3$ and $b = 4$. It will not matter which is which.

Since we are looking for the hypotenuse and have the two legs, we can set up and solve for $c$ (the hypotenuse) using the Pythagorean Theorem formula.

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$
The hypotenuse is 5 units long.

Notice that once we had the equation down to $25 = c^2$, we needed to square root both sides to undo the square.

Example 2: John puts the base of a 13-ft ladder 5 feet from the wall of his house. How far up the wall does the ladder reach?

Here, we have the hypotenuse and a leg. Since the hypotenuse must be $c$, let's assign $a = 5$ and $c = 13$. Now we can plug in and solve our formula.

\[
\begin{align*}
9 + 16 & = c^2 \\
25 & = c^2 \\
\sqrt{25} & = c^2 \\
5 & = c
\end{align*}
\]

The ladder goes 12 feet up the house.
Example 3: Find the length of the hypotenuse.

We have the legs, so \( a = 6 \) and \( b = 8 \). Now we can find \( c \) (our hypotenuse) using the Pythagorean Theorem.

\[
\begin{align*}
a^2 + b^2 &= c^2 \\
6^2 + 8^2 &= c^2 \\
100 &= c^2
\end{align*}
\]

If \( c = 10 \), then the hypotenuse is 10 units long.

Lesson Toolbox

Additional Resources and Readings

A video describing how to find the angles inside a triangle depending on which triangle you have
- Link to resource: https://www.youtube.com/watch?v=svjvBDA6xFQ

An instructional video that defines complementary and supplementary angles and shows how to tell where they are used and how to know the difference between them
- Link to resource: https://www.youtube.com/watch?v=GO20ZgUzlc0

An activity that works you through how to set up and solve Pythagorean Theorem problems
- Link to resource: https://kidsnumbers.com/math-games/pythagorean-theorem-madness/

Lesson Glossary

angle: formed by two rays that share a common endpoint; each ray is called a side of the angle
complementary angles: if the sum of the measures of two angles is 90°, then they are called complementary angles
hypotenuse: the side of the triangle opposite the 90° angle
perfect squares: the square of a whole number
Pythagorean Theorem: tells how the lengths of the three sides of a right triangle relate to each other; \( a^2 + b^2 = c^2 \) where \( c \) is hypotenuse and \( a \) and \( b \) are legs
ray: a line that has one endpoint
right triangle: a triangle that has one 90° angle
side of an angle: each ray of an angle
supplementary angles: if the sum of the measures of two angles is 180°, then they are called supplementary angles
triangle: a geometric figure with three sides and three angles
**vertex of an angle**: when two rays meet to form an angle, the common endpoint is called the vertex of the angle

**Check Your Knowledge**

1. Find the supplement of a 135° angle.
2. Find the complement of a 38° angle.
3. The measures of two angles of a triangle are 26° and 98°. Find the measure of the third angle.

**Answer Key:**
1. 45°  2. 52°  3. 56°

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