Measurements and Conversions

Inquire: Units of Measurement

Overview

If we want to know how long something is, how long it will take for something to happen, or how heavy
something is, we will need to measure. The measurements we commonly use, such as inches, seconds,
and grams, are called units of measurement. We can even combine units to give meaningful expression
such as, “Wow, that car is going 90 miles per hour! That is fast!” A useful skill is learning how to convert
from one unit to another. We will also learn how to convert between two systems of measurement: the
U.S. and metric systems.

By the end of this lesson, students will be able to convert within and between U.S. and metric unit
systems.

Big Question: How can I convert between units of measurement?

Watch: 525,600 Minutes

Did you know there are 525,600 minutes in a year? That is one way to measure a year!

How did we calculate such a huge amount of time? We can say that there are 60 minutes in 1 hour, 24
hours in 1 day, and 365 days in a typical year. By multiplying 60, 24, and 365, we can arrive at those
525,600 moments so dear.

In the United States and in scientific communities, we often use inches, cups, pounds, and minutes to
measure things such as length, volume, weight, and time. Within a type of measurement (such as time)
we can convert from minutes to hours to days and vice versa using conversion factors. These are well
known facts like there are 24 hours in a day.

In other places, they use the metric system. In this system, they pair prefixes with units to compare sizes
of measurements. An example of a metric unit is a meter, which is a measurement of length. The prefix
kilo- means thousand. This is why one kilometer is 1,000 meters.

We can also convert from one unit to the next within the metric system. Since the metric system is based
on multiples of ten, conversions involve multiplying and dividing by multiples of ten. It is a convenient
system that most of the world uses today.

Additionally, we have ways to convert between systems of measurement. This is usually done through
approximations. This means that while 12 inches will always be exactly 1 foot, 1 inch is only
approximately 2.56 centimeters. To keep things easy, we have decided that these measurements are good enough to convert between systems.

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for 22°C, what does that mean? This is a common example of needing to convert between systems. We can measure temperature in two ways: celsius and fahrenheit. Converting between the two involves understanding their relationship to each other.

The U.S. and metric systems use different scales to measure temperature. The U.S. system uses degrees Fahrenheit while the metric system uses degrees Celsius. Water boils at 100 degrees Celsius but at 212 degrees Fahrenheit. These degrees mean, and feel, the same.

Where do you see units of measurement used in your everyday life?

Read: U.S. and Metric Units

Overview

In this section, we will see how to convert among different types of units within the U.S and metric systems, such as feet to miles or kilograms to pounds. The basic idea with all of the unit conversions is to change the units, but not the value of a quantity, using multiplication.

U.S. Unit Conversions

There are two systems of measurement commonly used around the world. Most countries use the metric system. The United States uses a different system of measurement, usually called the U.S. system. We will look at the U.S. system first.

The U.S. system of measurement uses units of inch, foot, yard, and mile to measure length, and pound and ton to measure weight. For capacity, the units used are cup, pint, quart, and gallon. Both the U.S. system and the metric system measure time in seconds, minutes, or hours.

The equivalencies among the basic units of the U.S. system of measurement are listed in the table below. The table also shows, in parentheses, the common abbreviations for each measurement. We will refer to this table throughout this lesson.

<table>
<thead>
<tr>
<th>U.S. System Units</th>
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<tbody>
<tr>
<td><strong>Length</strong></td>
</tr>
<tr>
<td>1 foot (ft) = 12 inches (in)</td>
</tr>
<tr>
<td>1 yard (yd) = 3 feet (ft)</td>
</tr>
<tr>
<td>1 mile (mi) = 5280 feet (ft)</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
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<tr>
<td>Weight</td>
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<tr>
<td>------------------------------</td>
</tr>
<tr>
<td>1 pound (lb) = 16 ounces (oz)</td>
</tr>
<tr>
<td>1 ton = 2000 pounds (lb)</td>
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Suppose we want to convert inches to feet. We know that 1 foot is equal to 12 inches, so we can write 1 as the fraction $1 \text{ ft}/12 \text{ in}$. When we multiply by this fraction, we do not change the value, but do change the units. A fraction used to change the units of a measurement is called a **conversion factor**.

But 12 in/1 ft also equals 1. How can we decide whether to multiply by $1 \text{ ft}/12 \text{ in}$ or $12 \text{ in}/1 \text{ ft}$? We choose the fraction that will make the unit we want to convert from divide out. For example, suppose we wanted to convert 60 inches to feet. If we choose the fraction that has inches in the denominator, we can eliminate the inches unit.

\[
60 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 5 \text{ ft}
\]

\[
5 \text{ ft} \cdot \frac{12 \text{ in}}{1 \text{ ft}} = 60 \text{ in}
\]

Example 1: Mary Anne is 66 inches tall. What is her height in feet?

We will convert inches into feet by multiplying Mary Anne’s measurement and the conversion factor $1 \text{ ft}/12 \text{ in}$.

\[
66 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 66 \text{ ft}/12 = 5.5 \text{ feet}.
\]

Mary Anne is 5.5 feet tall.

Example 2: Ndula, an elephant at the San Diego Safari Park, weighs almost 3.2 tons. Convert her weight to pounds.

Since there are 2000 pounds in 1 ton, we will convert 3.2 tons into pounds using the conversion factor 2000 lbs/1 ton.

\[
3.2 \text{ tons} \cdot 2000 \text{ lbs/1 ton} = 3.2 \cdot 2000 \text{ lbs} = 6400 \text{ lbs}.
\]

Ndula weights almost 6400 pounds.

**Multiple U.S. Unit Conversions**

Sometimes to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.
Example 3: Juliet is going with her family to their summer home. She will be away for 9 weeks. Convert the time to hours.

To convert weeks into hours, we will first convert weeks to days and then days to hours. To do this, we will multiply each unit by the appropriate conversion factor. 7 days makes 1 week, and 24 hours are in 1 day.

Here is a way to visually represent the steps in this conversion as fractions.

\[
\frac{9 \text{ weeks}}{1} \times \frac{7 \text{ days}}{1 \text{ week}} \times \frac{24 \text{ hours}}{1 \text{ day}} = 1512 \text{ hours}
\]

Juliet will be away for 1,512 hours.

Example 4: How many cups are in 2 gallons of milk?

Use conversion factors to get the right units: convert 1 gallon to 4 quarts, 1 quart to 2 pints, and 1 pint to 2 cups.

Again, you can visually represent the steps in the conversion as fractions.

\[
\frac{2 \text{ gallons}}{1} \times \frac{4 \text{ quarts}}{1 \text{ gallon}} \times \frac{2 \text{ pints}}{1 \text{ quart}} \times \frac{2 \text{ cups}}{1 \text{ pint}} = 32 \text{ cups}
\]

There are 32 cups in 2 gallons of milk.

Metric Unit Conversions

Have you ever run a 5k or 10k race? The lengths of those races are measured in kilometers. The metric system is commonly used in the United States when talking about the length of a race, and in some other cases.

In the metric system, units are related by powers of 10. The root words of their names reflect this relation. For example, the basic unit for measuring length is a meter. One kilometer is 1000 meters; the prefix kilo- means thousand. One centimeter is 1/100 of a meter, because the prefix centi- means one one-hundredth (just like one cent is 1/100 of one dollar).

The equivalencies of measurements in the metric system are shown in the table below. The common abbreviations for each measurement are given in parentheses. Just like the U.S. conversions, we will refer to this table throughout this lesson.
<table>
<thead>
<tr>
<th>Metric Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
</tr>
<tr>
<td>1 kilometer (km) = 1000 m</td>
</tr>
<tr>
<td>1 hectometer (hm) = 100 m</td>
</tr>
<tr>
<td>1 dekameter (dam) = 10 m</td>
</tr>
<tr>
<td>1 meter (m) = 1 m</td>
</tr>
<tr>
<td>1 decimeter (dm) = 0.1 m</td>
</tr>
<tr>
<td>1 centimeter (cm) = 0.01 m</td>
</tr>
<tr>
<td>1 millimeter (mm) = 0.001 m</td>
</tr>
<tr>
<td>1 meter = 100 centimeters</td>
</tr>
<tr>
<td>1 meter = 1000 millimeters</td>
</tr>
</tbody>
</table>

To make conversions in the metric system, we will use the same technique we did with the U.S. system. We will multiply by a conversion factor of one to get to the correct units.

Example 5: Eleanor’s newborn baby weighed 3200 grams. How many kilograms did the baby weigh?

We will convert grams to kilograms knowing that 1 kilogram is 1000 grams.

\[3200 \text{ g} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 3.2 \text{ kg} \]

The baby weighs 3.2 kilograms.

**Using Decimals in Metric Conversions**

Since the metric system is based on multiples of ten, conversions involve multiplying by multiples of ten. We can simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1000, we move the decimal to the right 1, 2, or 3 places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left 1, 2, or 3 places, respectively. Do not forget to fill in zeros if you move the decimal over where there is not a number.

We can apply this pattern when we make measurement conversions in the metric system.

In Example 5, we changed 3200 grams to kilograms by multiplying by \( \frac{1}{1000} \) (or 0.001). This is the same as moving the decimal 3 places to the left.

\[3200 \cdot \frac{1}{1000} = 3.2\]
Example 6: Convert: A) 350 liters to kiloliters and B) 4.1 liters to milliliters.

From the table, 1 kiloliter = 1000 liters.

A) Since we are converting from liters to kiloliters, we will use the conversion factor 1 kL/1000 L. This means we will move the decimal place to the left 3 times. If we do that with 350 L, we will get 0.35 kL.

B) Since we are converting from kiloliters to liters, we will use the conversion factor 1000 L/1 kL. This means we will move the decimal place to the right 3 times. With the measurement 4.1 L, and filling in zeros when appropriate, we will get 4100 mL.

Reflect Poll: U.S. or Them?

Which system of measurement would you most likely use to measure how far it is from one city to the next?
- U.S. system
- Metric system

Expand: Converting Between Systems

Overview
In this section, we will focus on converting between U.S. and metric systems, and look toward understanding measurements in different dimensions.

Converting Between U.S. and Metric Systems
Many measurements in the United States are made in metric units. A drink may come in a 2-liter bottle, calcium may come in a 500-mg capsule, and we may run a 5k race. To work easily in both systems, we need to be able to convert between the two systems.

The table below shows some of the most common conversions.

<table>
<thead>
<tr>
<th>Conversion Factors Between U.S. and Metric Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
</tr>
<tr>
<td>1 in = 2.54 cm</td>
</tr>
<tr>
<td>1 ft = 0.305 m</td>
</tr>
<tr>
<td>1 yd = 0.914 m</td>
</tr>
<tr>
<td>1 mi = 1.61 km</td>
</tr>
<tr>
<td>1 m = 3.28 ft</td>
</tr>
</tbody>
</table>
We make conversions between the systems just as we do within the systems — by multiplying by unit conversion factors.

Example 1: Lee’s water bottle holds 500 mL of water. How many fluid ounces are in the bottle? Round to the nearest tenth of an ounce.

We should use the conversion factor that relates milliliters to ounces. There is 1 fluid ounce in 30 milliliters.

\[ 500 \text{ mL} \times \frac{1 \text{ fl oz}}{30 \text{ mL}} = \frac{500 \text{ fl oz}}{30} = 16.7 \text{ fl. oz.} \]

The water bottle holds 16.7 fluid ounces.

The conversion factors from system to system are not exact, but the approximations they give are close enough for everyday purposes. In Example 1, we rounded the number of fluid ounces to the nearest tenth.

Example 2: Soleil lives in Minnesota but often travels to Canada for work. While driving on a Canadian highway, she passes a sign that says the next rest stop is in 100 kilometers. How many miles until the next rest stop? Round your answer to the nearest mile.

There is 1 mile for about 1.61 kilometers.

\[ 100 \text{ km} \times \frac{1 \text{ mi}}{1.61 \text{ km}} = \frac{100 \text{ mi}}{1.61} \approx 62.1 \]

It is about 62 miles to the next rest stop.

Linear, Square, and Cubic Measure

When you measure your height or the length of a garden hose, you use a ruler or tape measure (see the figure below). A tape measure might remind you of a line — you use it for linear measure, which measures length. Inch, yard, mile, and meter are units of linear measure. Linear measure can be used when measuring how long an extension cord is or the length of the curtain rod.

![Tape Measure](image)

When you want to know how much tile is needed to cover a floor, or the size of a wall to be painted, you need to know the area, a measure of the region needed to cover a surface. Area is measured in square units. We often use square inches, square feet, square centimeters, or square miles to measure area. A square centimeter is a square that is one centimeter (cm) on each side. A square inch is a square that is one inch on each side. See the example below.
The figure below shows a rectangular rug that is 2 feet long by 3 feet wide. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.

When you measure how much it takes to fill a container, such as the amount of gasoline that can fit in a tank, or the amount of medicine in a syringe, you are measuring volume. Volume is measured in cubic units such as cubic inches or cubic centimeters. When measuring the volume of a rectangular solid, you measure how many cubes fill the container. We often use cubic centimeters, cubic inches, and cubic feet. A cubic centimeter is a cube that measures one centimeter on each side, while a cubic inch is a cube that measures one inch on each side. See the example below.

Suppose the cube in the figure below measures 3 inches on each side and is cut on the lines shown. How many little cubes does it contain? If we were to take the big cube apart, we would find 27 little cubes, with each one measuring one inch on all sides. So each little cube has a volume of 1 cubic inch, and the volume of the big cube is 27 cubic inches.
Lesson Toolbox

Additional Resources and Readings

A game of matching objects with units of measurement for the U.S. system
- Link to resource:
  http://www.sheppardsoftware.com/mathgames/measurement/BestMeasure2.htm

A game of matching objects with units of measurement for the metric system
- Link to resource:
  http://www.sheppardsoftware.com/mathgames/measurement/BestMeasureM.htm

A video describing a method for remembering the metric prefixes
- Link to resource: https://www.youtube.com/watch?v=5tHpDzXP-Ig

Lesson Glossary

area: the measure of the region, in square units, needed to cover a surface (e.g. square inches, square feet, square miles, etc.)
conversion factor: a fraction used to change the units of a measurement
linear measure: measurements that use a ruler (e.g. inch, foot, yard, mile, meter, etc.)
volume: the amount of space a liquid or object takes up in cubic units (e.g. cubic inches, cubic feet, cubic meters, etc.)

Check Your Knowledge

1. A floor tile is 2 feet wide. Convert the width to inches. (Include units written out)

2. Would you measure the height of a doorway using linear, square, or cubic units?
   a. cubic
   b. linear
   c. square

3. Would you measure the living area of an apartment using linear, square, or cubic units?
   a. cubic
   b. linear
   c. square

Answer Key:
1. 24 inches 2. B 3. C
Citations

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