Applications: Visually Modeling Fractions

Inquire: How to Model Fraction Operations

Overview

When learning, it can be said that we forget more when we focus on memorizing steps over making sense of our learning. This can apply to when students try to learn how to do the four basic operations with fractions. It is an important group of skills, but we are going to focus on modeling the fractions using common manipulatives. By the end of this lesson, students will use manipulatives (fraction bars or fraction circles) to model fractions and mixed numbers, add and subtract fractions with like and unlike denominators, and multiply and divide fractions.

Big Question: How can I model how to add, subtract, multiply, and divide fractions and mixed numbers?

Watch: The Ultimate Cookies

Let's admit it. Fractions are a challenging concept to understand for some. We try to learn them when we are still learning how to multiply and divide numbers. Since these are important skills to know when understanding fractions, we can have an issue right from the start.

If we do not have a firm understanding of what fractions look like, performing operations with them would be no easy task. We could even get by with memorizing what our teachers have taught us, but a lack of understanding stops us from really being able to use fractions as flexible numbers that relate parts to a whole. After a while, we might get by with using decimals instead and really miss out on the powerful use of fractions. We can't do away with them, so we need to go back to the basics. We need to create a concrete understanding of them using pictures. Don't think it is too late for you. Now is the time to go back and build a firm, fraction foundation.

Let's say you have a recipe for the ultimate chocolate chip cookie. The recipe calls for 3/4 cups of granulated sugar and 3/4 cups of packed brown sugar. How much sugar do you need altogether?

We could take out a piece of paper and pencil or pull out a calculator, but how about we try to visualize these fractions?
Let's imagine 2 fraction circles cut into fourths. That is eight-fourths altogether. If we fill in three-fourths for the granulated sugar and three-fourths for the brown sugar, how many fourths do we have? If you said six-fourths, you are correct. Six-fourths is the same as three-halves. We can also think of three-half cups of sugar as one and one-half cups of sugar. Those will taste like the ultimate chocolate chip cookie.

What if you need to make three batches of the ultimate chip cookies for a party? How many cups of granulated and brown sugar do you need for three batches of chocolate chip cookies?

This recipe is asking for three times as much sugar as last time. If we imagine 3 sets of three-halves, we will have nine-halves. If we do a little conversation, nine-halves is the same as four and one-half.

When do you think visualizing fractions will help you?
Read: Modeling Fractions and Operations

Overview

In this section, we will model proper fractions, improper fractions, and mixed numbers. Afterwards, we will focus on modeling operations with fractions. Try to relate how modeling matches the ways we add, subtract, multiply, and divide fractions.

Before we start, it may be useful to have your own fraction bars and circles to follow along. The following simulation should help out.

- Fraction Models from the National Council of Teacher of Math
  - https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Models

Play around with this simulator until you feel comfortable, then keep reading.

Modeling Proper Fractions, Improper Fractions, and Mixed Numbers

A fraction is a number that describes the parts of a whole. The top number of a fraction is the numerator and the bottom number of a fraction is the denominator. The denominator describes how many pieces a whole is divided into and the numerator describes how many parts to count out of that whole. We can use these definitions to model proper fractions which are fractions more than 0 but less than 1.

For the proper numbers, we will show you how to model the fractions using circles and bars.

Example 1: Model 1/2

1/2 can mean “one out of two,” “one-half,” and “one over two.” This looks like the following.

![Fraction Model 1/2](image)

Notice how each fraction is cut into two pieces (halves) and we have colored in 1 of those pieces. This matches the numerator and the denominator.

Example 2: Model 2/3

2/3 can be read as “two out of three,” “two-thirds,” and “two over three.” This looks like the following:
Two parts, out of the whole that has been cut in three pieces, have been filled in.

Let’s try modeling a mixed number. A **mixed number** is a number that has a whole number and a fraction. For the rest of the examples, we will alternate between circles and bars. Keep in mind that the concept is the same regardless of which modeling tool is used.

**Example 3: Model 1 \( \frac{3}{5} \)**

For this fraction, we will need two circles. One for the 1 and another for the 2/5. See this below:

[Image: Two circles, one showing 1 whole and the other showing 3 out of 5 pieces filled in.]

This picture shows that the whole number is a full circle (still cut into 5 pieces) and the fraction part has 3 out of 5 pieces filled in.

Finally, let’s model an improper fraction. An **improper fraction** is a fraction that has a bigger numerator than the denominator.

**Example 4: Model 8/5**

Even though the numerator is bigger than the denominator, the denominator still says our fraction is cut up in 5 pieces. This means we will need two bars as shown below.

[Image: Two bars, each showing 8 out of 5 pieces filled in.]
Compare the models for example 3 and 4. $1 \frac{3}{4}$ and $\frac{8}{5}$ are examples of equivalent fractions.

Fractions that have the same value, but different names are called equivalent fractions.

Modeling Adding Fractions with Like Denominators
We can model adding and subtracting fractions with like or same denominators.

Example 5: Model $\frac{1}{6} + \frac{2}{6}$
First, we need to model one-sixth and two-sixths.

If we count how many sixth slices we have, we can see that $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$, or three-sixths.

Example 6: Model $1 \frac{1}{4} + 2 \frac{3}{4}$
We will start by modeling each mixed number. For the whole part, we will use whole bars.

If we add the whole numbers, we have 3 wholes. If we add the fraction parts, we have 4 fourths. It takes four-fourths to make one whole, so that gives us another extra whole. This means that $1 \frac{1}{4} + 2 \frac{3}{4} = 4$.

Adding Fractions with Unlike Denominators
If we want to model adding and subtracting fractions with unlike denominators, we need to find equivalent fractions using the lowest common multiple (LCM) of the denominators (otherwise known as the lowest
common denominator or **LCD** of fractions). The rule is that we can only add fractions that have the same size pieces or denominator.

**Example 7: Add 1/2 and 1/3**

Since the fractions have unlike denominators, we need to find the LCD. Since the LCM of 2 and 3 is 6, we will find an equivalent fraction for 1/2 and 1/3 by breaking the pieces down into sixths. See this in the picture below:

We can see that 1/2 and 3/6 are equivalent because they are the same size. The same is true for 1/3 and 2/6.

Just like example 5 and 6, we can count our pieces to get 5 sixths. This means that 1/2 + 1/3 = 5/6.

**Example 8: Add 3/2 and 1 1/4.**

We are going to add an improper fraction to the mixed number. Let’s focus on looking for equivalent fractions for the fraction parts 3/2 and 1/4. We will come back to the 1 later.

The LCM of 2 and 4 is 4, so we will find an equivalent fraction for 3/2. We do not need an equivalent fraction for 1 1/4 since the denominator is already 4, but it will be modeling below:
Between 6/4 and 1 1/4, we have 11 fourths or 2 wholes and 3 fourths. This can mean we have $1 \frac{1}{4}$ or have $2 \frac{3}{4}$. These are both good solutions depending on your problem.

**Reflect Poll: Helping Modeling Operations**

After this lesson, which operation did modeling help you understand the most?
- adding and subtracting with like denominators
- adding and subtracting with unlike denominators
- multiplying fractions
- dividing fractions

**Expand: Modeling More Operations**

Modeling multiplication and division of fractions requires a bit more creativity, but it can be done by making sense of the fractions.

**Modeling Multiplying Fractions**

**Example 1: Multiply 3/4 by 1/2**

Three-fourths multiplied by one-half can be worded as the question “What is half of three-fourths?” By modeling three-fourths, we can begin to answer this question.

If we want to cut 3/4 in half, we need to cut each individual slice in half. This changes our model to this:
This gives us the equivalent fraction 6/8. If we only count half of these pieces, we have 3 eighth left. This means that \(\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}\).

**Example 2: Multiply 3/2 by 1/3**

Three-halves multiplied by one-third can be worded as the question “What is 1/3 of 3/2?” We can model 3/2 and then cut every piece into three equal pieces.

Now, we need to count only a third of the 9-sixth pieces we have. A third of 9 is 3, so multiplying the fractions together gets us 3-sixths.
Looking at 3/6, this looks exactly like 1/2. This means that $\frac{3}{2} \cdot \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$.

This is what it looks like when we can simplify a problem.

**Dividing Fractions**

A way to think about division is “how many of this number can go into that number?” For example, $\frac{12}{3} = 4$ because 3 can go into 12 four times. Let’s use this idea to help us divide fractions too.

**Example 3: Divide 1/2 by 1/4.**

So the question here is, “How many times can $\frac{1}{4}$ go into $\frac{1}{2}$?” It will be modeled below.

Another way to write $\frac{1}{2}$ is $\frac{2}{4}$. If there are 2 fourths inside of $\frac{1}{2}$, then $\frac{1}{2} ÷ \frac{1}{4} = 2$.

**Example 4: Divide 5/3 by 1/2**

The question here is, “How many one-halves go into five-thirds?” We can model this too.

We can count 3 halves, but what about the leftover piece. What can we call this?

If we are looking at the leftover piece, we can see that the half piece has cut the third piece in half. This leftover slice is one-sixth. This is modeled below.
The missing piece is 1/6.

Now we must ask ourselves how part of 1/2 is 1/6. Notice that 1/6 only fills the bottom half of the second circle a third of the way. With the 3-halves we got before, we can get that

$$\frac{5}{3} ÷ \frac{1}{2} = 3 \frac{1}{3}.$$

**Lesson Toolbox**

**Additional Resources and Readings**

A simulation allowing you to model fractions in the “fraction lab” or practice modeling proper fractions, improper fractions, and mixed numbers
- Link to resource: https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Fraction-Models/

A simulation allowing you to match fractions to their models
- Link to resource: https://phet.colorado.edu/en/simulation/fraction-matcher

A video describing how to divide fractions using squares on a grid
- Link to resource: https://www.youtube.com/watch?v=t1uazlu1Lu4

**Lesson Glossary**

- **fraction**: numbers that describe the parts of a whole
- **numerator**: the top number of a fraction; this describes how many parts to count out of the whole
- **denominator**: the bottom number of a fraction; this describes how many pieces a whole is divided into
- **proper fractions**: fractions bigger than 0 but less than 1
- **mixed number**: a number that has a whole number and a fraction
- **improper fraction**: a fraction that has a bigger numerator than the denominator
- **equivalent fractions**: fractions that have the same value, but different names
- **LCM**: lowest common multiple
- **LCD**: lowest common denominator

**Check Your Knowledge**

1. $\frac{1}{5} + \frac{2}{5} =$
   - a. $\frac{3}{10}$
   - b. $\frac{3}{5}$
   - c. $\frac{3}{2}$
   - d. $-\frac{1}{5}$
2. $\frac{1}{2} \cdot \frac{1}{4} =$
3. \( \frac{3}{8} + \frac{1}{4} = \)
   a. \( \frac{5}{2} \)
   b. \( \frac{3}{32} \)
   c. \( \frac{5}{8} \)
   d. \( \frac{3}{2} \)

Answer Key:

Citations

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