Fractions: Part 2

Inquire: Understanding Fractions

Overview

Understanding what a fraction is and looks like is the first step to performing addition, subtraction, multiplication, and division with them. Doing these operations allows us to form relationships between fractions and answer questions about how fractions work with each other. These problems might be a source of frustration for some, but looking at the problem with a strong understanding of fractions will help make sense of the operations. In this lesson, you will simplify, multiply, divide, add, and subtract proper and improper fractions.

Big Question: How can I add, subtract, multiply, and divide fractions?

Watch: Pizza and Fractions

It is easy to focus on the processes of adding, subtracting, multiplying, and dividing and miss making sense of the four operations. Before you start any operation, it is a good idea to make sense of the fractions.

Let’s do an example.

For a pizza party, you order three pizzas: cheese, pepperoni, and veggie. Each pizza is cut into 12 pieces. After the party, you notice that you have 5 pieces of cheese pizza, 3 pieces of pepperoni pizza, and 4 pieces of veggie pizza left. Will you be able to fit all the leftovers into one box?

Since each pizza was cut into 12 pieces, we can represent how much of each pizza is left using fractions. We could say that we have 5-twelfths cheese slices, 3-twelfths pepperoni slices, and 4-twelfths veggie slices.

If we are putting them all in the same box, that means we need to add the fractions together. Don’t take out the pencil and paper just yet. Recognize that each fraction has the same denominator, or slice size. If we add our fractions, we will have 12-twelfths.

Since twelve pieces makes one whole pizza, we have just enough space to fit our leftovers in one box.

This problem would be different if our pieces were cut in different sizes.

Let’s say we are at a different party with only pepperoni and veggie pizzas; and this time, the pepperoni pizza is cut into 6 pieces and the veggie pizza is cut into 12 pieces. If by the end of the party you only
have 3 slices of pepperoni pizza and 7 slices of veggie pizza left, will you be able to fit the leftovers into one box?

As fractions, we currently have 3-sixths of a pepperoni pizza and 7-twelfths of veggie pizza.

We still need to add our slices together, but it would not make sense to count the slices because they were not cut the same. If we had a common way to cut our slices, we could add our slices together.

You think that you can split the 3-sixth pepperoni pieces into 6 equal twelfth sized pieces. This would make each pepperoni piece the same size as the veggie pieces.

If you add the 6-twelfths of pepperoni slices with the 7-twelfths of veggie pizza, you will have 13-twelfths. You can only fit 12-twelfths of pizza in the box, so we cannot fit the leftovers in one box.

How do you think understanding fractions helps you do operations with them?

Read: Operations with Fractions

By the end of this lesson, we want to add, subtract, multiply, and divide proper and improper fractions. Keep in mind: methods for proper fractions work for improper fractions, and we will call them both “fractions” to make things easier.

If you can simplify fractions, it will make multiplying and dividing them much easier. We'll start our lesson here. Later, we will go over how to add and subtract mixed numbers (numbers with a whole and fraction part).

Simplify Fractions Using GCF

A simplified fraction is a fraction that has no common factors in the numerator and denominator. If you find the GCF (greatest common factor) between the numerator and denominator, you can divide the top and bottom number by it. This is called reducing, or simplifying, a fraction. This simplifies the fraction into an equivalent fraction with smaller numbers.

For example:

- \( \frac{2}{3} \) is simplified because there are no common factors of 2 and 3.
- \( \frac{10}{15} \) is not simplified because 5 is a common factor of 10 and 15.

Example 1: Simplify \( \frac{10}{15} \)

To simplify the fraction, we look for any common factors in the numerator and the denominator.

Since 5 is a common factor of 10 and 15, we can divide 10 and 15 by 5. This changes the fraction \( \frac{10}{15} \) into \( \frac{2}{3} \). Since there are no more common factors, this fraction is simplified.
Example 2: Simplify \( \frac{18}{12} \)
Let’s see what happens when we don’t simplify a fraction by its GCF.

2 is a common factor of 18 and 12. If we divide them both by 2, we have the fraction \( \frac{9}{6} \). This fraction is not simplified because 9 and 6 have a common factor of 3. If we divide out the 3 (or divide 6 in the original fraction), we will have the simplified fraction of \( \frac{3}{2} \) or 1 \( \frac{1}{2} \) (some fractions are more useful as mixed numbers).

Modeling Multiplying Fractions

Next, we will multiply fractions. A model may help you understand multiplication of fractions.

Example 3: Use fraction tiles to model \( \frac{1}{2} \cdot \frac{3}{4} \).

To multiply \( \frac{1}{2} \) and \( \frac{3}{4} \), think of “one half of three-fourths.”

Start with fraction tiles for three-fourths. To find one half of three-fourths, we need to divide them into two equal groups. Since we cannot divide the three \( \frac{1}{4} \) tiles evenly into two parts, we exchange them for smaller \( \frac{1}{8} \) tiles.

We see \( \frac{6}{8} \) is equivalent to \( \frac{3}{4} \). Taking half of the six \( \frac{1}{8} \) tiles gives us three \( \frac{1}{8} \) tiles, which is \( \frac{3}{8} \).

Therefore, \( \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \).

Multiplying Fractions with Simplification

We can also multiply across numerators and denominators. This will get us an equivalent fraction that we can simplify to get our answer. We can simplify before or after the multiplication.

Example 4: Multiply \( \frac{3}{5} \cdot \frac{2}{3} \)
To multiply these fractions, we will multiply 3 and 2 for the numerator and 5 and 3 for the denominator. This gets us the fraction \( \frac{6}{15} \). Dividing out the common factor of 3, we can get the simplified fraction \( \frac{2}{5} \).

We can simplify our fractions before using both fractions at the same time.
If we reuse Example 4, we can see if the numerators of both fractions have any common factors with the denominators of both fractions. We can see that 3 is the common factor between the numerator of $\frac{3}{5}$ and the denominator of $\frac{2}{3}$. This allows us to simplify the problem $\frac{3}{5} \cdot \frac{2}{3}$ into $\frac{1}{5} \cdot \frac{2}{1}$. If we multiply them across, we will get the answer $\frac{2}{5}$ (the same as before).

**Example 5: Multiply $\frac{3}{2} \cdot \frac{8}{9}$**

If we multiply across, we will get the fraction $\frac{24}{18}$. If we divide out the GCF of 6, we will have the fraction $\frac{4}{3}$.

If we try to simplify both fractions, we can divide a 3 out of 3 (numerator of the 1st fraction) and 9 (denominator of the 2nd fraction) and a 2 out of 2 (denominator of the 1st fraction) and 8 (numerator of the 2nd fraction). This changes $\frac{3}{2} \cdot \frac{8}{9}$ into $\frac{1}{1} \cdot \frac{4}{3} = \frac{4}{3}$, or $1 \frac{1}{3}$.

**Dividing Proper and Improper Fractions**

We need to use the reciprocal of a fraction to divide fractions. The **reciprocal of a fraction** is the fraction you get by flipping the numerator and denominator.

**Example 6: Divide $\frac{1}{2} \div \frac{1}{6}$ by modeling and changing the problem.**

We need to figure out how many $\frac{1}{6}$s there are in $\frac{1}{2}$.

![Diagram showing division of fractions](image)

Notice, there are three $\frac{1}{6}$ tiles in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{6} = 3$.

We can rewrite Example 6 into a multiplication problem by changing the sign and replacing the 2nd fraction with its reciprocal.

This changes $\frac{1}{2} \div \frac{1}{6}$ into $\frac{1}{2} \cdot \frac{6}{1}$. This gives us $\frac{6}{2}$ or 3 (same as before).

**Example 7: Divide $\frac{3}{8} \div \frac{9}{4}$ by changing it into a multiplication problem.**

If we change this problem by changing the sign and replacing the 2nd fraction with its reciprocal, we have an equivalent problem of $\frac{3}{8} \cdot \frac{4}{9}$. From here, you can multiply across or simplify. The simplified solution would be $\frac{1}{6}$.

This means $\frac{9}{4}$ will fit into $\frac{3}{8}$ one-sixth times. This may sound strange, but notice that $\frac{9}{4}$ is larger than $\frac{3}{8}$. It makes sense that you can only fit part of $\frac{9}{4}$ into $\frac{3}{8}$.
Add or Subtract Fractions with Common Denominators

If your fraction has common denominators, adding or subtracting fractions is straightforward. Let’s model fractions with common denominators.

Example 8: How many quarters are pictured, and what fraction of a dollar is this?

One quarter plus 2 quarters equals 3 quarters. Since a quarter is $\frac{1}{4}$ of a dollar, this model shows that $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ or 3 quarters.

Example 9: Subtract $\frac{4}{5} - \frac{1}{5}$.

The same idea for addition works for subtraction, too.

4 fifths with 1 fifth taken away leaves 3 fifths or $\frac{3}{5}$.

Example 10: Add $\frac{4}{9} + \frac{8}{9}$.

If you add 4 ninths to 8 ninths, you will have 12 ninths or $\frac{12}{9}$. We have an extra 3 ninths over 1, so we can simplify this fraction into $1 \frac{1}{3}$ or $\frac{4}{3}$ (sometimes an improper fraction is more useful for understanding).

Reflect Poll: Operations

Which fraction operation do you struggle with the most?

- Simplifying
- Multiplying
- Dividing
- Add or subtract with common denominator
- Add or subtract with uncommon denominator
Expand: Adding or Subtracting Fractions with Uncommon Denominators

With limited understanding, adding or subtracting fractions with uncommon denominators can become difficult; however, we can find and use the lowest common denominator (or LCD) to turn our fractions with uncommon denominators into equivalent fractions with common denominators. The **LCD of two fractions** is the least common multiple (LCM) of their denominators.

**Equivalent Fractions Using LCD**

**Example 1:** Change $\frac{1}{4}$ and $\frac{1}{6}$ to equivalent fractions with the LCD of 12.

Since the LCD is 12, we want to find out how many $\frac{1}{12}$ pieces fit into $\frac{1}{4}$ and $\frac{1}{6}$.

3-twelfths fit into 1-fourth, and 2-twelfths fit into 1-sixth. This means $\frac{1}{4} = \frac{3}{12}$ and $\frac{1}{6} = \frac{2}{12}$.

We can get this equivalent fraction by asking ourselves, “How many times does my denominator fit into the LCD?”

For 4, it fits into 12 three times; so, we need to break $\frac{1}{4}$ into three equal pieces. This means we should multiply the numerator and denominator by 3.

For 6, it fits into 12 two times; so, we need to break $\frac{1}{6}$ into two equal pieces. This means we should multiply the numerator and denominator by 2.

This will get us the same results as before: $\frac{1}{4} \cdot \frac{3}{3} = \frac{3}{12}$ and $\frac{1}{6} \cdot \frac{2}{2} = \frac{2}{12}$.

**Example 2:** Change $\frac{2}{3}$ and $\frac{1}{15}$ to equivalent fractions with the LCD of 15.

Notice that $\frac{1}{15}$ already has the denominator of 15. This means we only need to change $\frac{2}{3}$.

Since 3 fits into 15 five times, we need to multiply $\frac{2}{3}$ by $\frac{5}{5}$ to get the equivalent fraction of $\frac{10}{15}$.

**Changing Fractions**

Let’s get some practice adding and subtracting fractions with uncommon denominators.

**Steps to add or subtract fractions with different denominators:**
1. Find the LCD.
2. Convert each fraction to an equivalent form with the LCD as the denominator.
3. Add or subtract the fractions.
4. If needed, write the result in simplified form.

Example 3: \( \frac{1}{2} + \frac{1}{3} \)

First, we must get the LCD of the fractions which is the LCM of 2 and 3. Since 2 and 3 don’t have any common factors, we can multiply 2 and 3 together to get the LCD of 6.

Next, we will convert the fractions into their equivalent form using the LCD of 6.

- 2 fits into 6 three times, so \( \frac{1}{2} \cdot \frac{3}{3} = \frac{3}{6} \).
- 3 fits into 6 twice, so \( \frac{1}{3} \cdot \frac{2}{2} = \frac{2}{6} \).

Then, we will add the fractions. \( \frac{3}{6} + \frac{2}{6} = \frac{5}{6} \). This means \( \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \).

We do not need to simplify \( \frac{5}{6} \), but we should be careful of situations where we can.

Example 4: \( \frac{9}{10} - \frac{13}{20} \)

First, the LCD of the fractions is the LCM of 10 and 20. Using your preferred method, we find out that the LCD of the fractions is 20. This means that we do not need to change \( \frac{13}{20} \).

Next, we will convert \( \frac{9}{10} \) using the LCD of 20.

- 10 fits into 20 twice, so \( \frac{9}{10} \cdot \frac{2}{2} = \frac{18}{20} \).

Then, we will subtract our fractions \( \frac{18}{20} - \frac{13}{20} \) to get the fraction \( \frac{5}{20} \).

Notice that 5 and 20 have a common factor of 5. This means we can (and should) simplify \( \frac{5}{20} \) into \( \frac{1}{4} \).

After our work, we found that \( \frac{9}{10} - \frac{13}{20} = \frac{1}{4} \).

Common Pitfalls

Pitfall 1: Not finding a common denominator

If you had 1 quarter and 1 dime, you could say that you have 2 coins; but, when naming the coins, would it make sense to call them quarters? Dimes? Maybe quar-imes? Of course, the last is a joke, but that is what happens when students try to add \( \frac{1}{4} + \frac{1}{10} \) and get \( \frac{2}{14} \).
We can convert the value of a quarter and a dime into the common denomination of cents, or change the denominator to 100 (because there are one hundred cents in a dollar). This why \( \frac{1}{4} + \frac{1}{10} \) can be changed into \( \frac{25}{100} + \frac{10}{100} \) (meaning 25 cents + 10 cents). These add together to get \( \frac{35}{100} \) (meaning 35 cents).

Pitfall 2: Changing the denominator but not changing the numerator (when necessary)
Once you find the LCD, do not forget to multiply the top AND bottom of the fraction. This will make sure that our new fraction is equivalent to our old one.

\[
\begin{array}{c|c|c}
& \frac{1}{2} & \\
\hline
\frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

In the picture, we clearly see that we can break 1 half into 2 fourths.

A common mistake would be to find out 2 goes into 4 twice, then multiply \( \frac{1}{2} \) just the numerator or just the denominator by 2. This will get us the fractions \( \frac{2}{4} \) or \( \frac{1}{4} \). Both of these fractions are not the same size as \( \frac{1}{2} \).

Lesson Toolbox

Additional Resources and Readings
A game that practices modeling common fractions and equivalent fractions in circles. You must match all equivalent forms of a common fraction in a circle.

- Link to resource: https://www.mathplayground.com/Triplets/index.html

A visual game that has you match pictures of fractions with their equivalent ones. There are three levels of difficulty and a timed mode for added challenge.

- Link to resource: http://www.sheppardsoftware.com/mathgames/fractions/memory_equivalent1.htm

A game that flashes a picture and name of a fraction and you must select the correct square with the simplified fraction. You can increase the difficulty by making the grids larger or doing challenge mode.

- Link to resource: http://www.abcya.com/equivalent_fractions_bingo.htm

Lesson Glossary

simplified fraction: a fraction that has no common factors in the numerator and denominator
reciprocal of a fraction: the fraction you get by flipping the numerator and denominator of another fraction
LCD: lowest common denominator
LCD of two fractions: the least common multiple (LCM) of two fractions’ denominators
Check Your Knowledge

1. \( \frac{5}{9} \cdot \frac{3}{10} = \)
2. \( \frac{5}{9} + \frac{10}{12} = \)
3. \( \frac{5}{8} - \frac{7}{12} = \)

Answer Key:
1. \( \frac{1}{6} \)  2. \( \frac{2}{3} \)  3. \( \frac{1}{24} \)

Citations

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