GCF and LCM

Inquire: Factors and Multiples

Overview

Students continue to struggle with fractions as they make their way through math courses. This may come from a lack of understanding with what a fraction is and how to relate fractions to each other. A key concept is equivalent fractions, which is writing a fraction in multiple forms. The concepts of factors and multiples can help start to make sense of this important concept to fractions. In this lesson, we will determine if a number is divisible using divisibility tests and list all factors (or factor pairs) of a number. This will help us begin to write the lowest common multiple (LCM) and greatest common factor (GCF) of 2 or 3 numbers using the list or cake method.

Big Question: How can factors and multiples help us better understand fractions?

Watch: GCF and LCM

Greatest common factor (GCF) and least common multiple (LCM) are numbers that help us manage fractions better, but they can also help us when we have different groups of packaged items. We could either want to split them up evenly or have the same amount for each group.

Let’s consider a problem for each of these situations. In these problems, we are going to focus on the meaning of greatest common factor and lowest common multiple before getting into how to find them. We will start with a greatest common factor problem.

During the Christmas holidays, you want to make packages filled with baked cookies and brownies for your friends. You made 42 cookies and 24 brownies. You are interested in making the largest amount of packages possible, but you also want to make sure that everyone gets a fair, equal amount of cookies and brownies. How many cookies and brownies should you put in each care package so you have the most packages?

This is a situation is where greatest common factor is useful. With a little calculation, we find out that the GCF is 6. This means that we can make 6 packages with an equal share of cookies and brownies. If we divide 42 cookies and 24 brownies into 6 packages, that means everyone will get 7 cookies and 4 brownies. Sounds like a delicious solution!

How about a case where we use lowest common multiple?
You are trying to run a business that sells cheeseburgers. From your favorite vendor, you can buy cheese slices in packs of 40, buns in packs of 20, and burger patties in packs of 100. What is the lowest amount of cheese, buns, and burger packs you need to buy to have the same amount of each?

In this case, the LCM is 200. This means you need to buy enough packs to have 200 slices of cheese, buns, and burger patties each. If we divide each amount into 200, we need to buy 5 packs of cheese, 10 packs of buns, and 2 packs of burger patties. Are you getting hungry yet?

Can you think of a situation where you might be able to use GCF or LCM?

**Read: Multiples, Factors, and Division**

**Multiples** and **factors** can be useful to make it easier to add, subtract, and simplify fractions. **Division** connects the two ideas.

**Multiples**

Multiples are numbers we can get by multiplying a number by counting numbers. For example, the multiples of 2 are 2, 4, 6, 8, 10, 12, and so forth. We will find that patterns in multiples will create divisibility rules after we cover factors.

<table>
<thead>
<tr>
<th>Counting Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiples of 2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
</tr>
<tr>
<td>Multiples of 3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>Multiples of 4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
<td>44</td>
<td>48</td>
</tr>
<tr>
<td>Multiples of 5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
</tr>
<tr>
<td>Multiples of 6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
<td>66</td>
<td>72</td>
</tr>
<tr>
<td>Multiples of 7</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>42</td>
<td>49</td>
<td>56</td>
<td>63</td>
<td>70</td>
<td>77</td>
<td>84</td>
</tr>
<tr>
<td>Multiples of 8</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>56</td>
<td>64</td>
<td>72</td>
<td>80</td>
<td>88</td>
<td>96</td>
</tr>
<tr>
<td>Multiples of 9</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
<td>54</td>
<td>63</td>
<td>72</td>
<td>81</td>
<td>90</td>
<td>99</td>
<td>108</td>
</tr>
</tbody>
</table>

**Factors**

Factors are related to multiples by multiplication, but aren’t exactly the same thing. Factors are numbers that divide into numbers or multiply with another factor to get a number. This can be shown using “factor pairs” (or pairs of factors). One factor pair of 12 is 2 and 6, because 2 and 6 can divide 12 and 2 x 6 = 12. This makes 2 and 6 factors of 12. The rest of the factors of 12 are: 1, 12, 3, and 4. Notice that every factor has another factor to pair with. Sometimes a factor can pair with itself. For example, 5 is a factor of 25 because 5 x 5 = 25.

There are some numbers (bigger than 1) that only have one factor pair (1 and itself). Those are called “prime numbers.”
Now that we have discussed multiples and factors, let’s go over how to find them using divisibility rules for most numbers between 2-10.

**Divisibility Rule for 2: Even Numbers**

Sometimes it is useful to find factors of a number. We can do this by looking at patterns from multiples. For example, look at the multiples of 2 (table in the “Multiples” section). All the multiples of 2 are even numbers. This allows us to claim: “If it is an even number, then it is divisible by 2.” A few examples and counterexamples can show this is true:

\[
\begin{align*}
26/2 &= 13 & 188/2 &= 94 & 1,304/2 &= 652 & 2 \text{ is a factor of } 26, 188, \text{ and } 1,304 \\
27/2 &= 13.5 & 189/2 &= 94.5 & 1,305/2 &= 652.5 & 2 \text{ is NOT a factor of } 27, 189, \text{ and } 1,305
\end{align*}
\]

**Divisibility Rule for 5 and 10: Last Digit**

Let’s look at the last digits of the multiples of 5. Do you see a pattern?

You might notice that every multiple of 5 ends in either 0 or 5. This gives us the divisibility rule: “If the last digit is 0 or 5, then it is divisible by 5.” Let’s check this out:

\[
\begin{align*}
125/5 &= 25 & 890/5 &= 178 & 1,245/5 &= 249 & 5 \text{ is a factor of } 125, 890, \text{ and } 1,245 \\
384/5 &= 76.8 & 1,248/5 &= 249.6 & 5 \text{ is NOT a factor of } 384 \text{ and } 1,248
\end{align*}
\]

This pattern can also be used for the divisibility rule for 10: if the last digit is 0, then it is divisible by 10. Here are a few examples:

\[
\begin{align*}
90/10 &= 9 & 1,290/10 &= 129 & 343,800/10 &= 34,380 & 748/10 &= 74.8
\end{align*}
\]

This makes 10 a factor of 90, 1290, and 343,800 but not a factor of 748.

This idea of looking at the last digits can extend divisibility rules for more examples like 50, 100, and beyond.

**Divisibility Rule for 3 and 9: Summing Digits**

Some divisibility rules are not as obvious as others. For example, the divisibility rule for 3 is “If the sum of the digits are divisible by 3, then the number is divisible by 3.” The same goes for the divisibility rule for 9, but with 9 instead.

\[
\begin{align*}
87 \rightarrow 8 + 7 &= 15/3 = 5 & 123/3 \rightarrow 1 + 2 + 3 &= 6/3 = 2 & 655 \rightarrow 6 + 5 + 5 &= 16/3 = 5.3
\end{align*}
\]

This shows that 3 is a factor of 87 and 123, but not 655.

\[
\begin{align*}
936 \rightarrow 9 + 3 + 6 &= 18/9 = 2 & 1,253 \rightarrow 1 + 2 + 5 + 3 &= 11/9 = 1.2
\end{align*}
\]

This shows that 9 is a factor of 936 but not a factor of 1,253.
Divisibility Rule for 4 and 8: Halving

An alternative divisibility rule for 2 is “If a number can be halved, then it is divisible by 2.” Let’s extend this to claim, “If a number can be halved twice, then it is divisible by 4” and “If a number can be halved three times, then it is divisible by 8.”

160/2 = 80/2 = 40/2 = 20  This shows that 2, 4, and 8 are factors of 160.
124/2 = 62/2 = 31/2 = 10.5  This shows that 2 and 4 are factor of 124 but not 8.
814/2 = 407/2 = 203.5  This shows that 2 is a factor of 814 but not 4 and 8.
79/2 = 38.5  This shows that 2, 4, and 8 are not factors of 79.

Two extra takeaways from this:
1. Be aware there are alternative versions of the divisibility rules out there. You just need to find a pattern.
2. If it is NOT divisible by 2, then it is not divisible by 4 and 8. We can use divisibility rules to find if something is not a factor too.

Divisibility Rule for 6: Combining Rules

Divisibility rules can be combined to make rules for other numbers. Let’s take the divisibility rule for 6 for example. Since the only factors of 6 are 2 and 3, we can use them to create the rule, “If it is divisible by 2 and 3, then it is divisible by 6.”

96 is an even number and $9 + 6 = 15/3 = 5$, so 6 is a factor of 96.
74 is an even number but $7 + 4 = 11/3 = 3.6$, so 6 is NOT a factor of 74.
111 is not an even number but $1 + 1 + 1 = 3/3 = 1$, so 6 is NOT a factor of 111.

Combining divisibility rules can allow you be pretty creative with the rules we have discussed up to this point. The more rules you know, the easier it will be find factors and multiples of a number.

Divisibility Rule Recap

<table>
<thead>
<tr>
<th>A number is divisible by...</th>
<th>Rule:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>If it is an even number or can be halved</td>
</tr>
<tr>
<td>3</td>
<td>If the sum of the digits are divisible by 3</td>
</tr>
<tr>
<td>4</td>
<td>If a number can be halved twice</td>
</tr>
<tr>
<td>5</td>
<td>If the last digit is 0 or 5</td>
</tr>
<tr>
<td>6</td>
<td>If a number can be divisible by 2 and 3</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>8</th>
<th>If a number can be halved three times</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>If the sum of the digits are divisible by 9</td>
</tr>
<tr>
<td>10</td>
<td>If the last digit is 0</td>
</tr>
</tbody>
</table>

### Reflect Poll: Which Would You Use?

In your opinion, what part of this lesson do you think is most useful to you?

- divisibility rules
- multiples
- GCF
- LCM

### Expand: GCF and LCM two ways

One application of factors and multiples is finding greatest common factors (or **GCF**) and lowest common multiples (or **LCM**) between numbers. These can be used when adding, subtracting, or simplifying fractions and can be found by using the list or ladder method.

#### Finding GCF and LCM: List Method

The list method requires you to list every factor or enough multiples to find its GCF or LCM. See four examples below:

**Example 1: GCF of 12 and 18**

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 18: 1, 2, 6, 9, 18

12 and 18 have 1, 2, and 6 in common, so this makes 6 the greatest common factor.

**Example 2: LCM of 12 and 18**

Multiples of 12: 12, 24, 36

Multiples of 18: 18, 36

The first multiple 12 and 18 have in common is 36, so 36 is the LCM of 12 and 18.

**Example 3: GCF of 10 and 8**

Factors of 10: 1, 2, 5, 10

Factors of 8: 1, 2, 4, 8

10 and 8 only have 1 and 2 in common, so the GCF of 10 and 8 is 2.

**Example 4: LCM of 10 and 8**

Hot dogs are sold in packages of ten, but hot dog buns come in packs of eight. What is the smallest number of hot dogs and buns that can be purchased if you want to have the same number of hot dogs and buns?
This problem counts how many multiples of hot dogs and buns we need to have the same amount of both. Let’s list the multiples of 10 and 8.

Multiples of 10: 10, 20, 30, 40

Multiples of 8: 8, 16, 24, 32, 40

The first multiple of 10 and 8 in common is 40, so 40 is the LCM of 10 and 8.

One thing not to forget is that 1 can be the GCF of two numbers. If you list the factors of 2 and 7, the only number they have in common is 1. The GCF of 2 and 7 is 1.

Finding GCF and LCM: Cake Method

The cake method is useful when there are multiple numbers to find GCF and LCM, but it can be used for just two numbers.

Steps for the cake method:
1. Divide every number by a common factor (going big as possible will be fewer steps)
2. Write the number as step 1 and repeat until there are no more common factors.
3. Multiply all common factors. This is your GCF.
4. Multiply GCF by all leftover numbers. This is your LCM.

Example 5: GCF and LCM of 30 and 45

First, we will divide 30 and 45 by a number we know goes into 30 and 45. Let’s try 3. Because 30 divided by 3 is 10 and 45 divided by 3 is 15, we will write 10 and 15 above 30 and 45 respectively. Now we need a common factor between 10 and 15. We can try 5. After dividing 10 and 15 by 5, we will be left over with 2 and 3 which will be written on top of 10 and 15. There are no common factors for 2 and 3, so we will stop dividing.

GCF = 5 x 3 = 15

LCM = 2 x 3 x GCF = 6 x 15 = 90
When we multiply the 3 and 5 we divided out before, this gives us the GCF of 15. When we multiply the GCF by the leftover numbers (2 and 3), we get the LCM of 90.

Notice that 30 and 45 were both divisible by 15. The bigger the number you can divide out, the fewer the steps you need to find the GCF and LCM.

Example 6: GCF and LCM of 9, 12, 18

Between 9, 12, and 18, the only number we can divide out is 3. 9, 12, and 18 divided by 3 will be 3, 4, and 6. Since there are no common factors between these numbers, we will stop dividing here.

The GCF is 3 since that was the only common factor we could divide out of 9, 12, 18. We will get our LCM after we multiply our GCF (3) by 3, 4, and 6. This gets us a LCM of 216.

Lesson Toolbox

Additional Resources and Readings

A game allowing you to practice with LCM
- Link to resource: http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonMultiple.htm

A video focusing on recognizing situations to use LCM and GCF
- Link to resource: https://www.youtube.com/watch?v=zMkoldWRgUs

A video on how GCF is used to simplify fractions
- Link to resource: https://www.youtube.com/watch?v=2UZQ3MUNcHw

Lesson Glossary

multiples: numbers gotten by multiplying a number by counting numbers
factors: numbers that divide into numbers or multiply with another factor to get a number
prime numbers: numbers bigger than one with only one factor pair (1 and itself)
LCM: lowest common multiple
GCF: greatest common factor

Check Your Knowledge

1. Which of these is not a multiple of 3?
   a. 21
   b. 27
   c. 29
   d. 33

2. The given number is not divisible by which of these: 84?
   a. 2
   b. 3
   c. 4
   d. 5

3. Find the GCF and LCM of the given set of numbers: 24 and 30. (Write as GCF, LCM)

Answer Key:
1. C  2. D  3. 6, 120

Citations

Lesson Content:
Authored and curated by Kashuan Hopkins for The TEL Library. CC BY NC SA 4.0

Adapted Content:
Title: Prealgebra; 4.2 Multiply and Divide Fractions; Rice University, OpenStax CNX. License: CC BY 4.0
Link to resource: https://cnx.org/contents/yqV9q0HH@11.1:NjK_tuRw@15/Find-Multiples-and-Factors

Title: Prealgebra; 2.5 Prime Factorization and the Least Common Multiple; Rice University, OpenStax CNX. License: CC BY 4.0
Link to resource: https://cnx.org/contents/yqV9q0HH@11.1:BQ4z5ls4@21/Prime-Factorization-and-the-Le

Attributions

“Counting Number” By OpenStax is licensed under CC BY 4.0
Link to resource: https://cnx.org/contents/yqV9q0HH@11.1:NjK_tuRw@15/Find-Multiples-and-Factors

“Example 5” By TEL Library is licensed under CC BY NC SA
Link to resource: https://docs.google.com/document/d/1JSA4oGPZNA9v3agokgUITrmPBbuXo4bcfVBQbb6bzSg/edit?ts=5b719fa9#