Estimation

Inquire: When to Estimate?

Overview

You go out to celebrate with three other friends. Feeling in good spirits to see your friends, you generously agree to pay for everyone’s meal. You know that you and your friends have ordered four drinks for $1.50 each, a plate of nachos to share for $8.25, a chicken parmesan meal for $12.55, two salads for $8.95, and a burger for $11.25. You reach in your wallet and pull out $60. You wonder, is this enough money to cover the cost?

Estimation is a useful method for solving problems like this one where simple calculations can be used to solve problems mentally. In this lesson, you will use estimation strategies to check the reasonableness of an answer and improve mental calculations. Also, you will improve in mentally adding and subtracting integers.

Big Question: How and when can you use estimation to help perform quick mental calculations?

Watch: Footing the Bill

You are out with your good friends and feeling generous. You offer to pay the bill for the entire group. You realize a little too late that you only have $60 in your wallet and wonder if this will be enough the cover everything.

Let’s do the math.

You and your three friends have ordered four drinks at $1.50 a piece. That means you’ve spent around $2 per person. Since this is close to the exact amount, we can call this an estimation. We are rounding up, so we can even call this an overestimation. Since there are four drinks, you can multiply $2 by four people to know that you are spending around $8 on drinks.

Next is the nachos. As delicious as they are, the price tag of $8.25 is pretty close to $8. Our estimation is lower than our actual amount, so this means we have underestimated.

Now we have the main meals: a chicken parmesan meal for $12.55, two salads for $8.95, and a burger for $11.25. We can round the meals to $13 for the chicken, $9 for one salad, another $9 for the other salad, and $11 for the burger. That is around $42 for main dishes.

With the estimation we have done so far, we have the calculation: 8 + 8 + 42. If you already estimated $58, great work! That is a little over the exact amount of $55.95.
Looks like $60 will do it, but you might need a little help with tax and tip. Your friends, who are impressed you didn’t need a calculator, are more than willing to help you out.

Estimation is a powerful tool that can help simplify calculations we encounter in our everyday lives. It can be useful in cases like asking a cashier to retotal a bill when they say five bags of carrots selling at $1.85 a piece is worth $15 or you get the feeling you pressed the wrong button when you get “2,821” for $91 x 13 when you should have really gotten “1,183.” Even with a calculator, it is easy to make a mistake, so it is useful to check how reasonable our answers are with estimation.

What kinds of situations do you think would be made easier through estimation?

Read: Estimation and Integers

Why Estimate?

It is important to learn how to perform the four main operations well: addition, subtraction, multiplication, and division. This improves our ability to deal with integers. Integers are whole numbers that can be positive or negative, including zero (Example: ..., -2, -1, 0, 1, 2…).

![Integer Number Line](image)

While there are strategies that allow you to accurately calculate single digits, it is easier to make a mistake in calculation once the numbers get large. Consider the following problem: 5,012 x 995. Calculate this using pencil and paper or a calculator now.

If you got 4,986,940, great work! The traditional method is good, but it is easy to make a mistake. A calculator might be more efficient, but you must be careful not to push incorrect keys.

Introduction to Estimation

When appropriate, we can use estimation to help us avoid making mistakes. An estimation is a good guess at an exact calculation. You might notice that 5,012 is close to 5,000 and 995 is close to 1,000. If you round, the calculation that will get you “close enough” to the answer; 5,000 x 1,000 = 5,000,000 (count the zeros and multiply the whole numbers).

That is the power of estimation: to make a good guess at an exact answer, and check the reasonableness of calculations.

Place Value (Used for Rounding)

Rounding is a key skill used when estimating. It can be done by moving a selected digit up or down depending on its value and replacing the digits to the right of it with zeroes. A place value refers to a digit’s position in a number.
For example, see what 138 looks like.

138 can be broken down into a 1 in the hundred’s place, a 3 in the ten’s place, and an 8 in the one’s place. Every number can be broken down like this.

**How to Round**

Steps for Rounding:

1. Select a place value to round by.
2. If the place value is 1-4, then keep the digit the same.
3. If the place value is 5-9, then add one to the digit.
4. Replace the digits to the right of the place value with 0.

*The larger the place value you round by, the less accurate your estimation.*

**Zeroes to the right of the decimal do not need to be written.**

We typically round down or up depending on how close the digit is to 0 or 10. If the number is closer to 0 (like 1-4), then we keep the digit the same. If the number is closer to 10 (like 6-9), we round up.

To round 138 to the nearest tenth, we would see that 3 is in the ten’s place. Since the 8 is in the one’s place, and is in the range of 6-9, the ten’s place becomes 4. This gets us the rounded number 140.

If we round 138 to the nearest hundred, that would get us 100. We could even go to the nearest thousand, but that is just 0 (it is between 0 and 1,000).
Rounding: Addition

Let’s apply estimation to solve a few problems. Feel free to try the problems before reading the explanations:

1. Home Modeling: Sophia remodeled her kitchen and bought a new range, microwave, and dishwasher. The range cost $1,115, the microwave cost $275, and the dishwasher cost $525. About what was the total cost of these three appliances? What is the exact cost?

Try to round to the nearest hundredth on each amount and add them together.

The estimated cost of the applications to the nearest hundredth is $1,100, $300, and $500. The estimation is $1,900. That is not too far from the exact amount of $1,915.

Rounding: Accuracy

It is important to be careful how much we round. The bigger the place value we round by, the less accurate our guess is.

2. Shopping: A mountain bike is on sale for $393. Its regular price is $657. What is roughly the difference between the regular price and the sales price?

Let’s try rounding to the nearest ten’s place and the nearest hundred’s place.

Hundred’s place: $700 - $400 = $300
Ten’s place: $660 - $390 = $270
Exact: = $657 - $393 =  $264

Notice that the hundred’s place is the least accurate, but easiest calculation; while, the other calculations get harder but more accurate. It is a trade off you need to keep in mind when estimating.

Rounding: When to Use Underestimation and Overestimation

We can also apply estimation to multiplication and division.

3. Carpet: The living room in your house is rectangular with a length of 42 feet and a width of 34 feet. About how much carpet will you need to cover the area of the living room?

This is a case where you might want to overlook the rounding rules some, because it is important in this situation not to underestimate how much carpet you’ll need. Let’s underestimate and overestimate to get an idea of where the exact number will fall.

Underestimation: 40 x 30 = 1,200 square feet of carpet
Overestimation: 50 x 40 = 2,000 square feet of carpet

Our exact answer should be between 1,200 and 2,000 square feet.
Reflect Poll: When is it Useful?

In your opinion, when would estimation be most useful in helping you perform mental calculations?

- Buying expensive appliances
- Balancing checking and saving accounts
- Adding and subtracting positive and negative numbers
- Estimating how much gas money you need to fill up a tank for a car

Expand: Integers

Integers are useful when dealing with negative values. Estimation can be used in these situations as well.

Integers: Adding

1. Credit Cards: Frank owes $212 on his credit card. Then, he charges $105 more. What is the new balance?

Since Frank "owes" $212 and "charges" $105, the problem can be written like (-212) + (-105). We can use estimation to bring this to (-200) + (-100) = -$300. Frank’s credit card balance is about $300 into his limit. He owes exactly $317.

2. Football: The Rams took possession of the football on their own 35-yard line. In the next three plays, they lost 12 yards, gained 8 yards, then lost 6 yards. On what yard line was the ball at the end of these three plays?

Writing the following problem with integers looks like this: 35 + (-12) + 8 + (-6).
Rounding will have the problem look like this: 40 + (-10) + 10 + (-10) = 30.
We could estimate the ball is around the 30-yard line. The exact place is the 25-yard line.

Integers: Subtraction

3. Camping: Rene is on an Alpine hike. The temperature is −7°. Rene’s sleeping bag is rated "comfortable to −20°." How much can the temperature change before it is too cold for Rene’s sleeping bag?

The temperature is about −10°, while her sleeping bag is comfortable at about −20°. If we want to know the difference between these temperatures, the problem would look like -20 - (-10). This gives us -10°, or the temperature can drop about 10 degrees before it gets uncomfortable. The temperature can drop exactly 13 degrees before it gets uncomfortable, but it never hurts to be proactive on a hike.

4. Scuba Diving: Shelly’s scuba watch is guaranteed to be watertight to −100 feet. She is diving at −45 feet on the face of an underwater canyon. By how many feet can she change her depth before her watch is no longer guaranteed?

This is definitely a case where we do not want to overestimate. Shelly’s life could be in danger if she misjudges this. If we underestimate, that rounds the problem to -100 - (-50) = -50 feet. Shelly should be careful if she dives 50 feet beyond her current position. She has exactly 55 feet to go.
5. Temperature: On January 21, the high temperature in Palm Springs, California was 89°, and the high temperature in Whitefield, New Hampshire was −31°. What was the difference between the temperature in Palm Springs and the temperature in Whitefield?

Since we want to know the difference, we can round the problem to 90 - (-30). This gives us a good guess of 120° of difference between Palm Springs and Whitefield. The exact difference is 120°. Now that is a great guess!

Lesson Toolbox

Additional Resources and Readings

An article on places to use estimation
- Link to resource: https://www.mathsisfun.com/numbers/estimation.html?ref=driverlayer.com

An interactive soccer game allowing you to practice rounding and calculating by addition, subtraction, multiplication, or division
- Link to resource: http://www.abcya.com/estimating.htm

A pac-man style game for rounding and estimating in various situations, such as dollar amounts, the nearest thousand’s place, a number line, decimals, etc.
- Link to resource: http://www.sheppardsoftware.com/mathgames/menus/roundestimate.htm

Lesson Glossary

**integers**: whole numbers that can be positive or negative, including zero (...-2, -1, 0, 1, 2...)

**estimation**: a good guess at an exact calculation

**place value**: a digit’s position in a number

Check Your Knowledge

1. What is 172 rounded to the nearest ten’s place?

2. Round to the nearest tenth and calculate: 256 − 184.

3. Round to the nearest thousand and calculate: $6,251 + $4,749.

Answer Key:
1. 170  2. 260 - 180 = 80  3. 6,000 + 5,000 = $11,000