Calculating "OR" Probability

Inquire: “Or” Probability

Overview

When calculating compound probability problems for event A or event B, it is important to first determine whether the events are mutually exclusive. For events to be mutually exclusive, they can not occur at the same time. For example, you can not turn left and turn right at the same time. You can not be in San Francisco and New York City at the same time. These events are mutually exclusive. Events that are not mutually exclusive have an overlap. For example, if you are selecting one card from a deck, it is possible to choose an ace and a heart at the same time if you draw the ace of hearts. Therefore, the sets of aces and hearts are not mutually exclusive.

Big Question: When working with probability, why is it necessary to understand the differences between mutually exclusive and non-mutually exclusive?

Watch: Mutual Exclusivity

We make multiple choices and decisions every day. Sometimes we have good information to guide our choices, and sometimes we take our best guess and hope for the best. However, when calculating probability problems, we should have enough information to examine all possible outcomes before we choose.

When working with “OR” probability, we will have several decisions to make early in the process. Suppose there are 4 blue marbles, 5 red marbles, 1 green marble, and 2 black marbles in a bag. What is the probability that you will choose either a red marble or a blue marble on your first pick? This requires a calculation of an “OR” possibility. We are looking for an either/or outcome.

We must also consider mutual exclusivity. For all our attempts to multitask, there are times when two things can not occur at the same time. That’s the definition of mutually exclusive. For example, we can not run a race and take a bath at the same time. In calculating probability, it is often necessary to identify events as mutually exclusive or not before beginning to make calculations.

Calculation of “OR” probability offers two addition rules to follow. Non-mutually exclusive events follow the general rule of addition. Such calculations have an extra step to eliminate any overlap between subsets in the sample space. The extra benefit is that if you aren’t sure whether your events are mutually exclusive, you can always use the general rule of addition. It will work whether or not the events are mutually exclusive.
The addition rule for mutually exclusive events is a shorter equation since there is no possibility of overlap between events. The addition rule asks you to find the probability of each event and add them together. It will only work if there is no question about the two events being mutually exclusive, and should not be used without caution.

There are many questions that must be answered before any calculations begin. The calculations themselves are not the difficult part. Once you have identified the events, determined the exclusivity, and located the correct rule of addition, solving for the probability is a two or three step equation. Always gather as much information as possible before beginning any calculations.

Read: Calculating “OR” Probability

Overview

There are two types of compound probability. The first is the combination of outcomes from event A and event B. This is referred to as AND probability. The second is the outcomes from event A or B. This is referred to as OR probability. In calculating these outcomes from A or B, we must determine if the two events are mutually exclusive or non-mutually exclusive. These questions must be answered before any calculations are done because different formulas are needed depending on such information. This lesson will focus on OR probabilities and their calculations.

Mutually Exclusive or Not

A and B are mutually exclusive events if they cannot occur at the same time. This means that A and B do not share any outcomes and \( P(A \text{ AND } B) = 0 \).

For example, suppose the sample space \( S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \).
Let \( A = \{1, 2, 3, 4, 5\} \)
Let \( B = \{4, 5, 6, 7, 8\} \)
Let \( C = \{7, 9\} \)

\( A \) and \( C \) do not have any numbers in common so \( P(A \text{ and } C) = 0 \). Therefore, \( A \) and \( C \) are mutually exclusive.

\( A \) and \( B = \{4, 5\} \). These are the terms that overlap set \( A \) and set \( B \).
\( P(A \text{ and } B) = 2/10 \) There are 2 outcomes out of 10 possible outcomes. It is not equal to zero.
Therefore, \( A \) and \( B \) are not mutually exclusive.

The following Venn diagram illustrates a similar example of non-mutually exclusive events. There is an overlap between \( A \) and \( B \). To have been mutually exclusive, the circles for \( A \) and \( B \) could have no common elements.
There are slightly different rules for calculating probability for events that are mutually exclusive and those that are not.

**Addition Rule (For Mutually Exclusive Events)**

The addition rule for compound probability is when two events, A and B, are mutually exclusive, the probability that A or B will occur is the sum of the probability of each event.

For example, using a well shuffled deck of cards, pick a card. What is the probability that it will be the seven of spades? There is only one seven of spades in a standard deck of 52 cards. The probability is 1/52. P(seven of spades) = 1/52

What is the probability of drawing a heart? There are 13 hearts in a deck, so the probability is 13/52. P(heart) = 13/52

What is the probability of drawing a seven of spades or a heart? These are mutually exclusive events. It is impossible to draw a heart that is also the seven of spades. Therefore, we use the formula for addition which is:

$$P(A \text{ or } B) = P(A) + P(B)$$

P(seven of spades or a heart) = 1/52 + 13/52 = 14/52 = 26.9%.

Using the same deck of cards, what is the probability of drawing a king or a queen? There are four kings and four queens in each deck. The probability of each is 4/52.

P(K) = 4/52

P(Q) = 4/52

P(K or Q) = 4/52 + 4/52 = 8/52 = 15.4%

These examples are mutually exclusive events. They cannot occur at the same time.

**General Addition Rule (For Non-Mutually Exclusive Events)**

A second addition rule exists for events which are non-mutually exclusive. When two events, A and B, are non-mutually exclusive, there is some overlap between them. The formula becomes: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. The last part of this formula - $P(A \text{ and } B)$ allows us to subtract the elements that are in both A and B. Let's look at a Venn diagram again.

Suppose an experiment has the outcomes 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Let event $A = \{1, 2, 3, 4, 5, 6\}$.
Let event $B = \{6, 7, 8, 9\}$. 
Then \( A \) AND \( B = \{6\} \). The number 6 is contained in both \( A \) and \( B \).

And \( A \) OR \( B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \).

Using the formula \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \)

\[
= \frac{6}{12} + \frac{4}{12} - \frac{1}{12} \\
= \frac{10}{12} - \frac{1}{12} \\
= \frac{9}{12} = 75\%
\]

There is a 75% probability that the numbers in the sample space will be elements in \( A \) or \( B \). You can verify this by counting the numbers in \( A \) or \( B \).

Let's do another example with the deck of cards. What are the probabilities for the following draws?

Let \( A = P(\text{King}) = \frac{4}{52} \)

Let \( B = P(\text{Club}) = \frac{13}{52} \)

Let \( C = P(\text{King of Clubs}) = \frac{1}{52} \)

In this example, \( A \) and \( B \) are not mutually exclusive because the king of clubs is both a king and a club. It is a member of \( A \) and \( B \).

What is the probability of drawing a king or a club?

\[
P(A \text{ or } B) = P(A) + P(B) - P(A + B) \\
= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \\
= \frac{16}{52} = 30.8\%
\]

Reflect Poll: Mutually Exclusive Events

Which example of mutually exclusive events helped you most to understand its use in probability?

- events modeled on Venn diagrams
- examples with marbles
- examples with cards
- example of married and single people

Expand: Calculating Without a Venn

Overview

While Venn diagrams are helpful in visualizing probabilities, they aren't essential. Take the following problem for example. If we were given a diagram, we would see two circles that overlap. We immediately know that the events are not mutually exclusive and calculating the number of outcomes must use the general rule of addition. In order to not count some of the sample twice, the elements of the overlap must be subtracted.
Example: French and Spanish
Suppose we have a class of 30 students. 16 are studying French and 21 are studying Spanish.
Let b = how many study both languages. If we were looking at a Venn diagram of this example, the
overlap between the group studying French and the group studying Spanish would be the group labeled b.

Students studying French only = 16 - b. This is the number of students studying French minus the number
studying French and Spanish.

Students studying Spanish only = 21 - b. This is the number of students studying Spanish minus the
number studying Spanish and French.

Since we know the total number in the group is 30, we can solve for b. (16−b) + b + (21−b) = 30. This is
the number studying French only, plus the number studying both, plus the number studying Spanish is
equal to the total group.

37 - b = 30
b = 7

Now that we know the totals, we can figure the following probabilities:

P(French) = 16/30
P(Spanish) = 21/30
P(French Only) = 9/30
P(Spanish Only) = 14/30
P(French or Spanish) = 30/30 = 1
P(French and Spanish) = 7/30

Since we know this example is not mutually exclusive, we can check our probabilities by using the
general addition formula.

P(A or B) = P(A) + P(B) − P(A and B)
= 16/30 + 21/30 - 7/30
= 37/30 - 7/30
= 30/30 = 1
There were no outcomes outside the groups of French or Spanish, and all of the outcomes were within the two groups. The probability of a student studying either French or Spanish is 1, or 100%. It is a certainty that all of the students are in one of the two groups.

Lesson Toolbox

Additional Resources and Readings

A quick lesson on finding the probabilities of mutually exclusive events
- Link to resource: https://youtu.be/jDnU3-jX4GA

A video showing mutually exclusive events and non-mutually exclusive events, and the addition formulas for each one
- Link to resource: https://youtu.be/LPeWICWWPhU

A video showing how to find the probabilities of mutually exclusive and non-mutually exclusive events using the sample space of a deck of cards and the sample space of rolling two dice
- Link to resource: https://youtu.be/RJ0ihN6qsGQ

An excellent video explaining in detail how to do problems with mutually and non-mutually exclusive events
- Link to resource: https://youtu.be/gzYk192apRw

Lesson Glossary

**mutually exclusive**: A and B are mutually exclusive events if they can not occur at the same time

**non-mutually exclusive**: A and B are non-mutually exclusive if they can occur at the same time

**AND probability**: a type of compound probability calculating the combination of outcomes from event A and event B

**OR probability**: a type of compound probability calculating the combination of outcomes from event A or event B

Check Your Knowledge

1. Which of the following in a deck of cards is mutually exclusive?
   - a. kings and diamonds
   - b. hearts and diamonds
   - c. clubs and aces
   - d. spades and jacks

2. Which of these could not be part of a Venn diagram?
   - a. circles
   - b. frequency table
   - c. overlap of circles or ovals
   - d. elements outside the circles

3. Mutually exclusive events can occur at the same time.
   - a. True
   - b. False
Answer Key:

Citations

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